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From Arithmetic to Algebra: Students' Epistemological Obstacles

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ABSTRAK

Konsep awal aljabar yang pertama kali di pelajari siswa adalah persamaan linier satu variabel, dan konsep persamaan linier satu variabel yang pertama kali menjadi kesulitan yang dialami oleh siswa adalah memahami makna dari variabel. Penelitian ini bertujuan untuk mengkaji hambatan epistemologis yang dialami siswa pada konsep persamaan linier satu variabel. Penelitian ini menggunakan pendekatan kualitatif fenomenologi. Partisipan pada penelitian ini adalah 35 orang siswa kelas VIII Sekolah Mengengah Pertama yang sudah mempelajari konsep persamaan linier satu variabel. Pada tahap awal penelitian, peneliti mengembangkan dua buah permasalahan berupa soal uraian, pemilihan soal uraian dalam penelitian ini bertujuan untuk mengukur proses mental siswa dalam menuangkan ide atau gagasan ke dalam jawaban. Selanjutnya peneliti melakukan wawancara kepada 4 orang siswa untuk memberikan penguatan terhadap judgment peneliti terhadap jawaban-jawaban yang sudah diberikan siswa terkait soal uraian. Berdasarkan hasil dan pembahasan didapatkan bahwa kurangnya pemaknaan siswa dalam proses pembelajaran menyebabkan munculnya hambatan belajar epistemologis. Hambatan belajar ini diidentifikasi melalui solusi yang diajukan oleh siswa yang sebagian besar bersifat aritmetika daripada aljabar, meskipun siswa mengakui bahwa permasalahan yang dihadapi berkaitan dengan konsep persamaan linear satu variabel, mayoritas siswa tetap mengalami kesulitan dalam memberikan solusi dalam bentuk persamaan linear satu variabel, khususnya dalam bentuk $ax \pm b = cx$ dan $ax \pm b = cx \pm d$.

ABSTRACT

The initial algebraic concept typically introduced to students is the linear equation in one variable. One of the earliest difficulties students encounter with this concept is comprehending the significance of a variable. This study aims to investigate the epistemological obstacles faced by students in acquiring proficiency in linear equations in one variable. A qualitative phenomenological methodology was employed in this research. The participants consisted of 35 eighth-grade students enrolled in a junior high school who had previously studied linear equations in one variable. In the initial phase of the study, the researcher formulated two word problems. The utilization of word problems was intended to elucidate students' thought processes in articulating their ideas or reasoning in written form. Subsequently, interviews were conducted with four selected students to corroborate the researcher's interpretation of their written responses. The findings and discussion reveal that the conceptual deficiencies acquired during instruction contributed to the emergence of epistemological learning obstacles among students. This learning obstacle was identified through the solutions proposed by students, which were predominantly arithmetic rather than algebraic. Although the students acknowledged that the problem they were dealing with was related to the concept of linear equations in one variable, the majority still encountered difficulties in providing solutions in the form of a linear equation in one variable., particularly in the forms $ax \pm b = cx$ and $ax \pm b = cx \pm d$.

INTRODUCTION

One of the primary objectives of education is to empower students to attain their full academic and professional potential. To achieve this, a robust foundational understanding, particularly in mathematics, is indispensable. Algebra holds significant importance during the cognitive development of students from elementary school to junior high school. Regrettably, numerous students develop persistent misconceptions regarding algebraic concepts, which can impede their ability to realize this potential (Zielinski, 2017). One contributing factor is that students (and, in some instances, even teachers) tend to prioritize memorizing and applying rules without comprehending the underlying reasoning behind them, known as instrumental understanding. This approach contrasts with developing a deeper comprehension of mathematical structures and understanding the appropriate actions and reasons, referred to as relational understanding (Skemp, 2020). In accordance with this perspective, Lundberg & Kilhamn (2018) underscore the importance of teachers not only focusing on solving textbook problems and providing explanations but also considering the intended learning outcomes from these tasks. In accordance with this, Yıldız & Yetkin Özdemir (2021) assert that teachers often hold the misconception that students can comprehend a number pattern after acquiring algebraic expressions, rather than the reverse. This erroneous belief may result in missed opportunities for students to transition from comprehending the pattern's structure to interpreting letters in algebra as varying quantities, ultimately leading to the notion of generalization in algebraic expressions.

Algebra is a fundamental subject for students as they transition into secondary education. The initial algebraic concept typically introduced at this level is the linear equation in one variable. One of the initial challenge students encounter is comprehending the significance of the variable itself (Maudy et al., 2017; Lucariello et al., 2014). Understanding variables is paramount, as it serves as the foundation for students to perform more advanced algebraic operations. As (Tall, 1992) suggests, algebraic symbols often function as both processes and concepts, emphasizing the utmost importance of comprehension. Consequently, to facilitate students' comprehensive comprehension of algebraic concepts, an instructional approach that serves as a connecting link is imperative.

A commonly adopted approach by students in interpreting and utilizing algebraic letters is rooted in logical reasoning and prior experience. This often reflects their attempts to comprehend unfamiliar symbols or notations. Many eleven-year-old students with no prior exposure to algebra tend to perceive these letters as abbreviations. For instance, they may interpret the letter "b" as referring to "Brian" and the letter "f" as standing for "fruit" (Stacey & MacGregor, 1997). Wahyuni et al. (2023) conducted research that revealed that students' interpretations of solving problems involving algebraic letters still centered on the difficulty of converting notations into natural numbers. Notably, seventh-grade students are typically expected to have had prior learning experiences in algebra, including an understanding of algebraic letters as generalized numbers. This issue warrants attention from teachers and mathematics education experts in developing and implementing algebra learning materials, particularly concerning linear equations

Linear equations constitute a cornerstone of mathematics instruction, particularly at the lower secondary level. Although linear equations do not arise naturally, they are embedded within a diverse range of problems in algebra, geometry, and trigonometry (Hall, 2002). Research has also demonstrated that linear equations, particularly those involving one variable, present unique learning obstacles for students (Jupri et al., 2014). These obstacles often stem from students' perception that algebraic problems possess a single, fixed representation. Additionally, a lack of understanding of the underlying meaning behind algebraic expressions leads to difficulties in recognizing equivalent algebraic forms (Dahlan & Juandi, 2011).

Furthermore, the transition from arithmetic to algebra presents a significant learning obstacle. Even students who excel in arithmetic may encounter difficulties in algebra due to the shift in cognitive processes required for algebraic thinking, which diverges substantially from their prior understanding (Herscovics & Linchevski, 1994). Additionally, students encountered obstacles in translating word problems into mathematical expressions, applying the concept of place value, executing addition and subtraction calculations. This is attributed to the limited capacity of teachers to present problems with a diverse range of approaches (Sidik et al., 2021). If these learning obstacles are not addressed with due seriousness, they can accumulate and exacerbate obstacles as students progress to more advanced algebraic concepts. Additionally, when designing instructional tasks for linear equations, teachers must consider both the structural and functional relationships involved. This encompasses the development of arithmetic-to-algebraic representation skills, their application to represent more intricate problems, solving linear equations, and addressing contextual problems that necessitate a deeper conceptual grasp (Suryadi et al., 2023). In light of these considerations, this study endeavors to investigate the epistemological obstacles faced by students in comprehending the concept of linear equations in one variable.

METHOD

This qualitative study employed a phenomenological approach. Qualitative research involves the active participation of the researcher in both data collection and analysis processes, with the researcher serving as the primary instrument throughout the entire research process (Creswell, 2012). The researcher is responsible for focusing on all aspects of the study, including identifying data sources, collecting and analyzing data, and drawing conclusions based on field findings. The participants in this study were 35 eighth-grade students from a junior high school who had previously studied the concept of linear equations in one variable.

In the initial phase of the study, the researcher developed two word problems. The utilization of word problems served as a means to explore students' mental processes in expressing their ideas and reasoning through written responses. These word problems were administered to students who had already studied the topic of linear equations, with the primary objective of assessing the extent of their comprehension of the concept. Furthermore, the assessment was conducted to identify potential learning obstacles encountered by students, encompassing misconceptions, procedural errors, and difficulties in comprehending algebraic structure.

Subsequently, the researcher conducted interviews with four students, whom the researcher deemed to represent the overall responses of the 35 students in the class. The purpose of conducting interviews with these four students was to further validate the students' answers and to strengthen the researcher's interpretation of the students' responses to the word problems presented. The interview methodology employed in this study was semi-structured interviews, which struck a balance between flexibility in eliciting information and maintaining focus on the discussion topics.

The data analysis in this study was conducted using a qualitative approach, adhering to the stages adapted from Miles & Huberman (1994), which include:

1. Data reduction: Students' written responses and interview results were selected, focused, and simplified to identify key information related to epistemological learning obstacles.
2. Data display: The data was presented in the form of descriptive narratives to provide a comprehensive overview of students' thinking patterns and the types of difficulties they encountered.
3. Conclusion drawing and verification: This involved interpreting the displayed data and ensuring that the findings were consistent with field data through triangulation between students' written responses and interview data.

RESULT AND DISCUSSION

Based on the results of tests conducted on 35 junior high school students, the researcher subsequently conducted further analysis to explore the students' responses in greater detail. The researcher conducted interviews with four students who were representative of the learning obstacles experienced by the other 35 students. Learning obstacles refer to challenges or difficulties encountered by students in comprehending a particular learning concept. (Brousseau, 2002) identified three types of learning obstacles: ontogenical learning obstacles, epistemological learning obstacles, and didactical learning obstacles. This study focuses on the analysis of epistemological learning obstacles experienced by students in relation to the concept of linear equations in one variable.

The researcher presented two distinct problems pertaining to the concept of linear equations in one variable. Each problem was meticulously crafted to elucidate students' learning obstacles associated with this mathematical concept.

The initial problem was designed to identify potential learning obstacles associated with linear equations in the forms of $x \pm b = c$ and $ax \pm b = c$. The problem is presented as follows:

> *Dino received a saving shaped like a chicken from his father. As a result, Dino became motivated to save money at home. Almost every day, he set aside a portion of his pocket money to put into the saving. After one year, Dino planned to open the saving. However, just before doing so, he added Rp. 5,000. After opening and counting the contents, he found that his total savings amounted to Rp. 477,500. How much money had Dino saved before he added the last Rp. 5,000? Try to solve this problem using at least two different methods.*

From the initial problem, the researcher distinguished three distinct types of student responses, each indicative of distinct learning obstacles in comprehending the concept of linear equations in one variable.

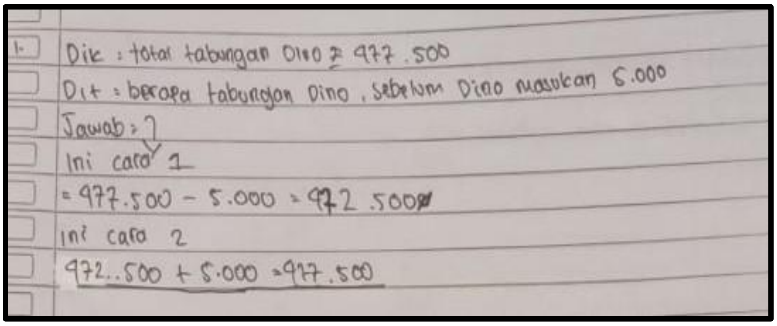


Figure 1. Variations in Student 1's Responses to Problem 1

Student 1 successfully provided a correct solution to the given problem and even proposed two alternative methods of resolution. In the first approach, the student directly performed a subtraction operation based on the information provided in the problem: $477.500 - 5.000 = 472.500$. In the second approach, the student applied an addition operation to the result obtained from the first step: $477.500 - 5.000 = 472.500$. These two alternative solutions demonstrate that the student possessed a procedural understanding of the fundamental quantitative relationships among the given numbers. However, it is evident from both responses that the student was unable to construct a formal or symbolic mathematical model in the form of a linear equation in one variable, such as $x + 5.000 = 477.500$. This suggests that the student was still operating within a concrete and arithmetic mode of thinking and had not yet reached the structural stage of reasoning that characterizes algebraic understanding. Further exploration through interviews revealed that the student recognized the problem as related to the topic of linear equations in one variable, yet lacked sufficient comprehension of how to construct a formal representation of the given situation. The student expressed confusion when asked to formulate the equation using a variable. This difficulty reflects the presence of an epistemological obstacle: the student's inability to comprehend the concept of a variable as a symbol that represents an unknown value within a mathematical relationship. Such an understanding is foundational to the transition from arithmetic to algebra, and the absence of this awareness underscores the conceptual obstacles that learners often encounter at this critical juncture.

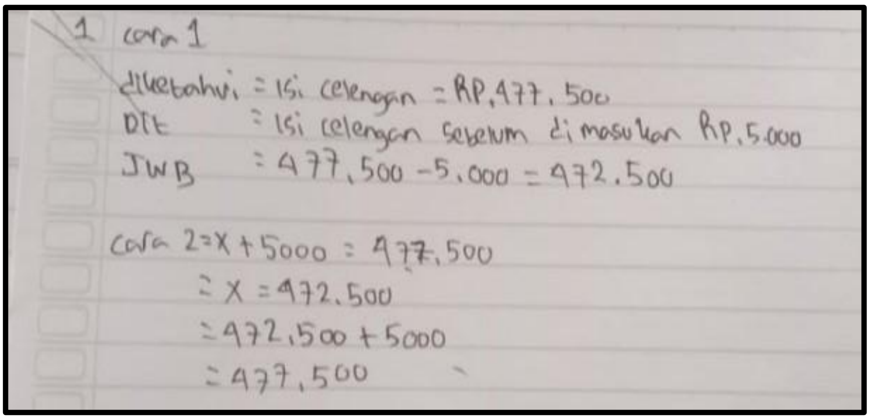


Figure 2. Variations in Student 2's Responses to Problem 1

As depicted in Figure 2, Student 2 successfully resolved the provided problem accurately. The student also exhibited two distinct solution approaches: the initial approach involved a subtraction procedure, while the second approach transformed the problem into a formal mathematical equation. Student 2 demonstrated proficiency in utilizing a variable (x) to represent the unknown amount of money. At this juncture, the student 2 exhibited a clear comprehension of the concept of a linear equation in one variable, specifically in the form $x \pm b = c$, and was capable of solving it. However, despite the correctness of both the model and solution, it appears that the student continues to encounter obtacles in the domain of formal representation, particularly with regard to the proper utilization of mathematical notation and adherence to conventional structural conventions. The student 2 has not yet demonstrated a clear distinction between the steps required to solve an equation and the steps necessary to verify the solution. Initially, the student 2 correctly wrote the equation after obtaining the answer from the first method. However, they immediately substituted the value of x in the equation with the correct answer. While this method may be correct if the student 2 possesses accurate algebraic knowledge, it is important to note that the student used an arithmetic approach in the first method. Therefore, it is necessary to conduct a deeper investigation to understand why the student directly substituted the value of x with the actual result.

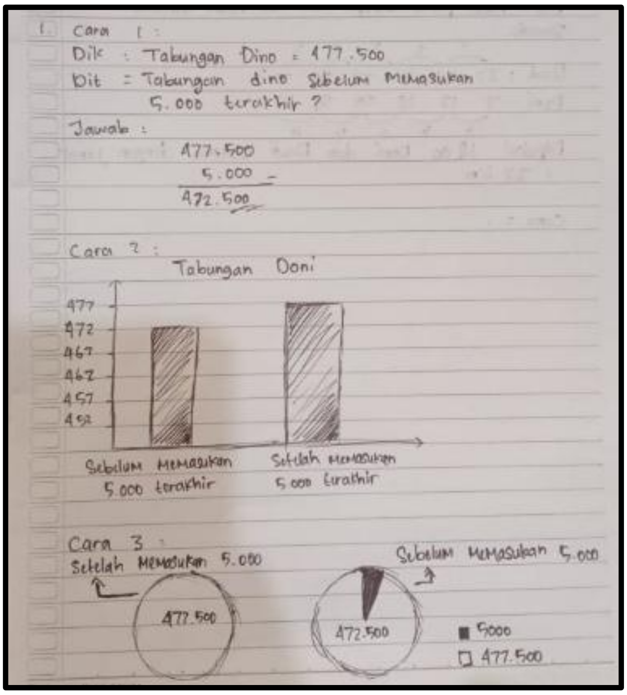


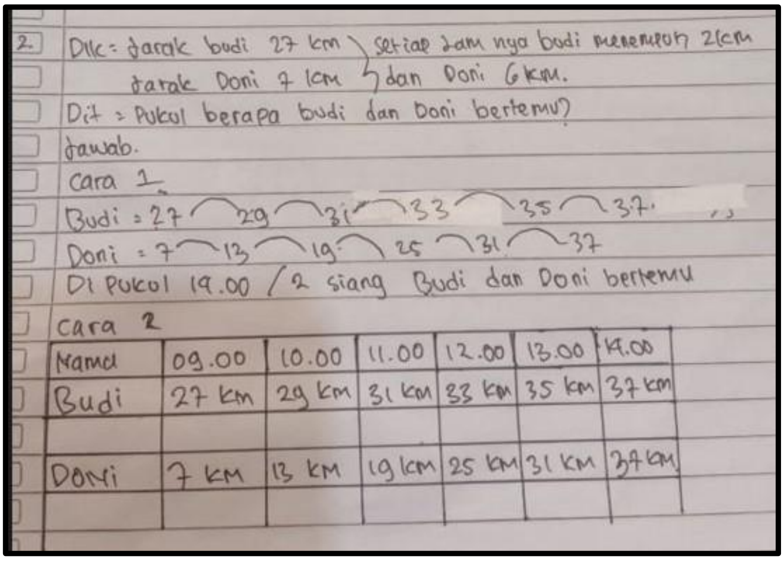
Figure 3. Variations in Student 3's Responses to Problem 1

As depicted in Figure 3, Student 3 successfully resolved the provided problem. For the initial solution, akin to Students 1 and 2, the student employed a subtraction-based approach to solve the problem. However, for the second solution, Student 3 adopted a distinctive method by transforming the problem into visual representations, specifically a bar diagram and a pie chart. his suggests that Student 3 demonstrates proficient problem-representation skills. While they successfully depicted the mathematical situation using distinct representations from other peers, it is apparent that Student 3 did not explicitly express the problem as a mathematical equation. Interviews conducted with Student 3 revealed that they acknowledged the potential of representing the problem as an equation but opted to solve it employing arithmetic techniques and diagrams.

The second problem was designed to identify students' learning obstacles related to the concept of linear equations in one variable, expressed as $ax \pm b = cx$ and $ax \pm b = cx \pm d$. The problem is as follows:

Budi and Doni are traveling from Surabaya to Jakarta by car, departing at 9:00 a.m. Budi has already traveled 27 kilometers, while Doni has only traveled 7 kilometers. Budi drives at a speed of 2 km/h, and Doni drives at a speed of 6 km/h. At what time will Budi and Doni meet along the way? Try to solve the problem using at least two different methods.

In the second problem, the researcher identified two distinct types of student responses, which unveiled various learning obstacles pertaining to linear equations in one variable.


2. Dik = Jarak budi 27 km } setiap jam nya budi menempuh 2 km
jarak Doni 7 km } dan Doni 6 km.
Dit = Pukul berapa budi dan Doni bertemu?
Jawab.
Cara 1
Budi = 27 29 31 33 35 37
Doni = 7 13 19 25 31 37
Di Pukul 14.00 / 2 siang Budi dan Doni bertemu
Cara 2

Nama	09.00	10.00	11.00	12.00	13.00	14.00
Budi	27 km	29 km	31 km	33 km	35 km	37 km
Doni	7 km	13 km	19 km	25 km	31 km	37 km

Figure 4. Variations in Student 1's Responses to Problem 2

As depicted in Figure 4, Student 1 successfully arrived at the correct solution to the provided problem and exhibited flexibility in problem-solving by employing two distinct methods: a sequential numerical approach and a tabular format. Although both strategies relied on repeated addition based on the distances covered per unit of time, these methods reflect the student's operational understanding of the problem context. Specifically, the student systematically computed the accumulated distances traveled by Budi and Doni over time, enabling them to determine the point at which their distances became equal. This suggests an initial form of relational reasoning, albeit still rooted in arithmetic thinking. Nevertheless, the student was not yet capable of constructing or utilizing a formal algebraic equation to represent the problem, which is fundamentally related to linear equations in one variable. The absence of a symbolic representation, such as the equation $27 + 2x = 7 + 6x$, while the student demonstrates procedural and contextual reasoning skills, indicates a lack of transition to algebraic modes of thinking. This reflects the presence of an epistemological obstacle, particularly in the transition from arithmetic-based representations to symbolic-algebraic representations, an obstacle commonly encountered during the early stages of algebraic learning.

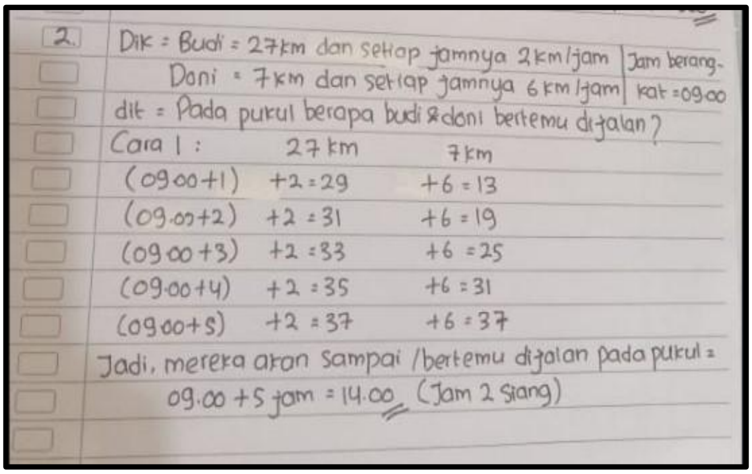


Figure 5. Student 4's Response to Problem 2

As depicted in Figure 5, the student successfully provided the correct solution to Problem 2. Notably, the solution presented by Student 4 bears a striking resemblance to that of Student 1. This similarity can be attributed to Student 4’s reliance on an enumerative approach, wherein time and distances traveled by both Budi and Doni are incrementally added.

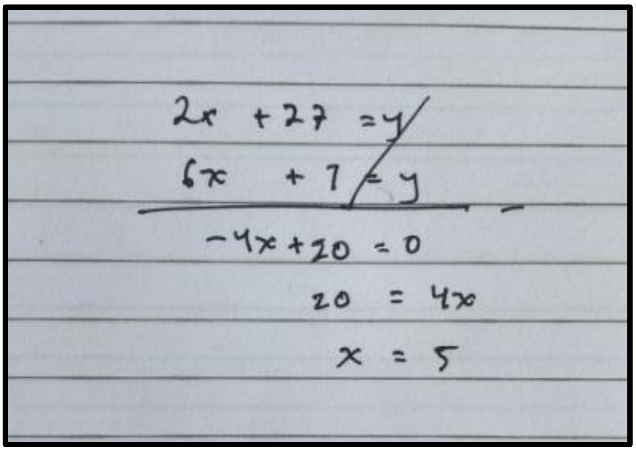


Figure 6. Student 5's Response to Problem 2

As depicted in Figure 6, student 5 exhibited a reasonably sound level of procedural algebraic proficiency, as evidenced by their ability to construct and solve a system of linear equations through appropriate symbolic manipulation. The elimination process employed to equate the two equations and solve for the variable (x) was executed correctly from a technical perspective. However, a substantial conceptual error was evident in the identification and selection of the appropriate mathematical model, particularly in the context of a linear equation in one variable (LEOV). This error suggests that, while the student 5 has acquired proficiency in symbolic manipulation techniques, their conceptual comprehension of the structure and limitations of LEOV remains undeveloped. The student 5 failed to distinguish between a system of linear equations in two variables and linear equation in one variable.

In this problem, students 5 are tasked with expressing the given situation in the form of an equation, where a and b are constants, and x is the variable. The equation should be in the form $ax \pm b = cx \pm d$, where $a, b, c, and\ d$ are integers. The objective of this problem is to determine the value of x that makes the expressions on both sides of the equation equal. Furthermore, the student 5 has not yet demonstrated the capacity to connect symbolic representations with the contextual meaning of the problem being addressed. This reflects an ongoing obstacle in bridging the gap between formal symbolic reasoning and contextual understanding, which serves as a key indicator of epistemological obstacles in algebraic learning. Consequently, while the student 5 appears competent in procedural aspects, their conceptual grasp of the fundamental essence and intended function of linear equations in one variable remains in the developmental stage.

Out of the 35 participating students, the majority were able to provide answers or solutions to the two problems presented by the researcher. However, most students approached both problems using counting strategies rather than employing formal mathematical symbols or expressions. This tendency may be attributed to the position of algebra as a subsequent stage in the mathematical curriculum following arithmetic, which often results in students having a greater familiarity with and ease in interpreting arithmetic expressions compared to algebraic ones (Sarımanoğlu, 2019).

This pattern also suggests the existence of a conceptual gap between arithmetic and algebra, arising from a shift in students' cognitive orientation from viewing mathematics as a set of procedures (processes) to perceiving it as abstract structures (objects). Consequently, students are required to develop a dual process-product perception to navigate this transition effectively (Sfard & Linchevski, 1994). Furthermore, the researcher identified a student who attempted to solve the second problem by formulating it as a system of linear equations in two variables. Although the provided answer was correct, and the student was able to express it in a mathematical sentence, the result suggests that the student has not yet fully grasped the process of accurately translating a problem into a suitable mathematical statement.

The diverse student responses suggest the presence of epistemological learning obtacles in students' comprehension of the presented problem and their ability to articulate it mathematically as a linear equation in one variable. These obstacles are reflected in students' difficulties in associating the given problems with relevant mathematical concepts specifically, the concept of linear equations in one variable, which led to the emergence of epistemological obstacles (Suryadi, 2019). The observed epistemological obstacles suggest that students are not only grappling with procedural proficiency in symbolic manipulation, but more fundamentally, they are encountering difficulties in interpreting and assigning significance to algebraic expressions. This misalignment between students' conceptual comprehension and the abstract nature of algebra necessitates a re-evaluation of how algebraic thinking is cultivated in the initial stages of mathematics education.

Based on the interview process conducted, the researcher ascertained whether students comprehended the problem presented and the mathematical concept it pertained to. The students acknowledged the problem's association with linear equations in one variable. However, they encountered obstacles in applying this concept. This difficulty stemmed from the transition of knowledge from arithmetic to algebra, which posed initial obstacles during algebra learning and persisted as students progressed to more advanced levels (Maudy et al., 2017). Concurrently, students' proficiency in executing fundamental algebraic algorithms is crucial for developing an understanding of the mathematical entities involved. Furthermore, a conceptual grasp of these entities is essential for achieving complete technical competence, particularly in assigning significance to algebraic algorithms (van Amerom, 2002). Students currently in the eighth grade of junior high school continued to grapple with this transition, resorting to arithmetic-based solutions such as counting. The lack of engaging and meaningful lessons provided by the teacher contributed to these difficulties, underscoring the necessity for more effective learning methodologies (Suryadi, 2019).

Radford (2022) stated that by constructing a series of progressively complex story problems and subsequently asking students to create and write their own stories which are subsequently translated and solved using the symbolic system they have acquired this approach will enhance students' awareness and comprehension of the conceptual structure of relational equations, as well as the operational rules involved in simplifying them. Furthermore, Kieran et al. (2016) proposed an alternative approach to introducing algebra in elementary school, suggesting that it be taught as early algebra. The primary objective of early algebra instruction is to equip students with the ability to identify patterns and regularities. Additionally, it aims to foster their capacity to formulate, test, and justify rules or conjectures through collaborative classroom interactions involving small groups or whole-class discussions. This process ultimately enables students to construct knowledge based on their own ideas. This process can serve as a robust foundation for secondary school students to grasp algebra, facilitating a seamless transition from arithmetic to algebra.

To support this approach more effectively, specific strategies are needed to assist teachers in overcoming various learning obstacles encountered by students**.** In Hall's (2002) study, Koehler and Grouws identified four strategies to mitigate students' learning obstacles:

1) Analyzing students' errors provides valuable insights for developing teaching strategies that anticipate future errors.
2) Analyzing students' errors elucidates the development of their initial thinking processes during the learning process.
3) Utilizing misconceptions and common errors as discussion points in relevant sections of the solution model requires caution to avoid confusion.
4) Teachers can compare specific student errors with those listed to gain a deeper understanding of the nature of students' mistakes.

Furthermore, the integration of technology in algebra instruction has been demonstrated to enhance students' engagement and foster the development of various learning characteristics (Alsaeed, 2017). Consequently, the utilization of technology not only enhances students' learning experiences but also contributes to making algebra instruction more meaningful and contextually relevant to students' needs.

CONCLUSION

Based on the results and discussion, it was determined that the lack of substantial comprehension among students during the classroom learning process leads to the emergence of epistemological learning obstacles. These learning obstacles were identified through the solutions proposed by the students, which were predominantly arithmetic in nature rather than algebraic. Although the students acknowledged that the problems they encountered were related to the concept of linear equations in one variable, they still encountered difficulties in providing solutions in the form of a linear equation with one variable, particularly in the forms $ax \pm b = cx$ and $ax \pm b = cx \pm d$. To address these obstacles, an alternative instructional approach proposes introducing algebra from the early years of schooling as a means of fostering students' capacity to discern patterns and regularities. Through collaborative learning, whether in small groups or whole-class discussions, students are encouraged to formulate, test, and justify rules or conjectures based on their own reasoning. Furthermore, by constructing a series of progressively more intricate story problems and prompting students to create and solve their own problems utilizing algebraic symbols they have acquired, learners are gradually led to a deeper comprehension of the relational structure of equations and the logical principles governing their simplification. This approach facilitates students in developing algebraic understanding in a meaningful and incremental manner, grounded in context and rooted in their own conceptualizations.

Consequently, analyzing the diverse errors or learning difficulties encountered by students not only offers valuable insights for the refinement of instructional practices but also facilitates teachers' ability to anticipate the recurrence of similar errors or obstacles. This empowers teachers to gain a deeper comprehension of students' thought processes during the learning journey. By examining these learning obstacles, teachers can utilize students' misconceptions, prevalent errors, and learning obstacles as discussion points in designing instructional approaches for the subject of linear equations. Furthermore, by employing appropriate technology and drawing upon the analysis of students' epistemological learning obstacles, teachers can devise instructional strategies that effectively mitigate the emergence of such obstacles, particularly in the context of linear equations in one variable. Ultimately, these endeavors contribute to the realization of learning that is more meaningful, contextual, and aligned with students' cognitive abilities and learning preferences.

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